EXHIBIT F

TO RULE 4.2 STATEMENT OF DR. DOUGHERTY

Design, and Applications of Fin Lines, Bharathi Bhat and
ı K. Koul
'ntegrated Circuits, P. Bhartia and P. Pramanick, eds.
'th Helical and Folded Helical Resonators, Peter Vizmuller
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CAPACITANCE, INDUCTANCE, AND CROSSTALK ANALYSIS

Charles S. Walker

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$$Z_{02} = \sqrt{\frac{L_2}{C_2}}\ \Omega \qquad (36)$$

and

$$\sqrt{\frac{(L_2/l)(C_2/l)}{(L_1/l)(C_1/l)}} = \sqrt{\frac{\mu\varepsilon}{\mu\varepsilon}} = 1 \qquad (37)$$

Substituting Eqs. (35), (36) and (37) into Eq. (32) yields Eq. (29):

$$C_m = \frac{L_m}{Z_{01}Z_{02}}\ \text{F/m}$$

REFERENCES

1. Boast, William B., *Principles of Electric and Magnetic Fields*, New York, Harper and Brothers, 1956, pp. 205–210, 229, 311.
2. Mohr, R.J., "Coupling between Open Wires over a Ground Plane," *IEEE Symp. on EMC*, July 23–25, 1968, pp. 404–413.

2.2.4 Capacitance between Parallel, Vertical, Flat Conductors, C-4

This formula set introduces the concept of fringing flux. This is an important consideration because fringing increases the value of the capacitance between parallel vertical conductors above that calculated neglecting fringing, often by many times. Please see Formula Set C-6 for the derivation of the fringing factor. An important application is the determination of the capacitance between lands on printed wiring boards such as those shown in Fig. 2.10.

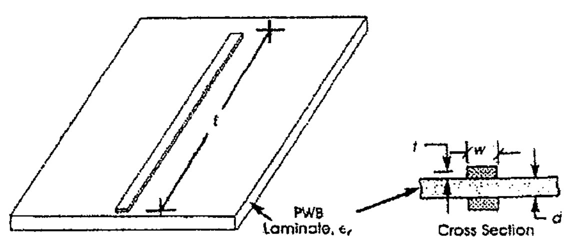

Figure 2.10 Parallel, flat conductors are located on opposite sides of a PWB at a distance d apart.

Equations:

The value of capacitance per unit length is defined as

$$\frac{C}{l} = \varepsilon_r \varepsilon_0 K_{C1}\left(\frac{w}{d}\right)\ \text{F/m} \qquad (1)$$

$$= 8.84\ \varepsilon_r K_{C1}\left(\frac{w}{d}\right)\ \text{pF/m} \qquad (2a)$$

$$= 0.225\ \varepsilon_r K_{C1}\left(\frac{w}{d}\right)\ \text{pF/in} \qquad (2b)$$

where K_{C1} = Capacitive fringing factor (1 or greater). For $d/w \ll 1$, $K_{C1} = 1$.

Fringing Flux:

As previously noted, flux fringing increases the value of the capacitance. Figure 2.1 illustrates this idea with a sketch plotting the approximate flux patterns for a paralle plate capacitor with and without fringing.

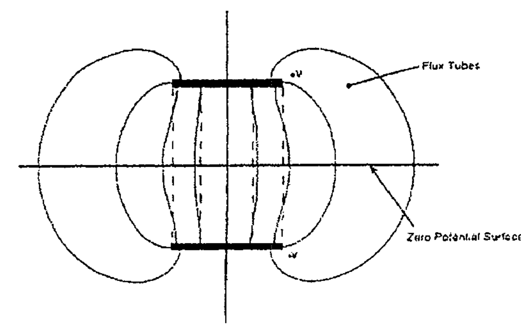

Figure 2.11 The dimensions for this parallel plate capacitor are chosen to show a capacitance increase due to fringing of 2.5 times. Flux lines without fringing are shown dashed. This sketch of the flux pattern shows a total of 10 flux tubes (*versus* 4 in the dashed line case). Thus the capacitance is 2.5 times greater. (The surrounding medium has a relative dielectric constant, $\varepsilon_r = 1$ and is thus homogeneous for this specific case.)

Example:

Determine the capacitance between two vertical conductors as shown in Fig. 2.10 with these dimensions:

$t = 0.0028''$ (2 oz. copper) $\quad d = 0.060''$

$w = 0.025'' \quad l = 6''$

Assume that a printed wiring board with epoxy-glass laminate with a relative dielectric constant $\varepsilon_r = 4.5$.

Step 1: Using Fig. 2.12, determine the fringing factor, K_{C1}, for $d/w = 0.060''/0.025'' = 2.4$.

Step 2: Determine the capacitance from Eq. (2b) above.

$$C = 0.225\,\varepsilon_r K_{C1}\left(\frac{w}{d}\right) \times l \text{ pF}$$

$$= 0.225 \times 4.5 \times 2.4 \times \left(\frac{0.025''}{0.060''}\right) \times 6'' \text{ pF}$$

$$= 6.1 \text{ pF} \qquad (3)$$

The measured value for this example was 6.4 pF showing excellent correlation.

Derivation:

(Please refer to Formula Set C-6.)

Commentary and Conclusions:

1. This formula set graphically illustrates the effect of fringing on "parallel" plate capacitors. Figure 2.12 indicates that for $d/w = 16$, the actual capacitance is 7.9 times *greater* than would be predicted from direct parallel plate equations, neglecting fringing. This increase could mean the difference between achieving or not meeting the crosstalk requirements.
2. In most PWB designs the land width dimensions, w, are about the same or smaller than the thickness of the circuit boards. For example, for 0.060″ thick board and a trace with $w = 0.020''$, $d/w = 3$ (which is not $\ll 1$). Thus, electric flux fringing cannot be ignored.

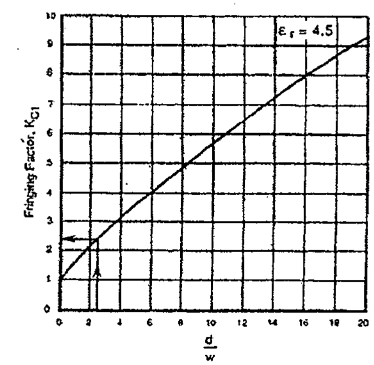

Figure 2.12 The fringing factor, K_{C1}, is determined by entering the graph at the ratio d/w, proceeding upward to the curve and locating K on the left-hand vertical axis as indicated. Thus, for $d/w = 2.4$, $K_{C1} = 2.4$. (This figure was developed in Formula Set C-6.)

2.2.5 Capacitance between Horizontal Flat Conductors, C-5

The capacitance value between horizontal, rectangular conductors is important because this configuration closely approximates conductor lands on printed wiring boards such as those shown in Fig. 2.13.

Equations:

The capacitance per unit length is given approximately by

$$\frac{C}{l} \approx \frac{\pi \varepsilon_{r(eff)} \varepsilon_0}{\ln\left(\dfrac{\pi(d-w)}{w+t} + 1\right)} \text{ F/m} \qquad (1)$$

$$= \frac{27.8\,\varepsilon_{r(eff)}}{\ln\left(\dfrac{\pi(d-w)}{w+t} + 1\right)} \text{ pF/m} \qquad (2a)$$

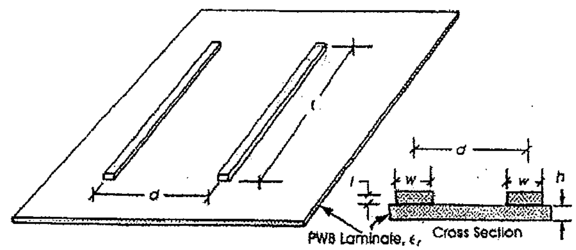

Figure 2.13 Two rectangular conductors are located distance d apart on the top of a circuit board with thickness h and relative dielectric constant, ε_r.

$$= \frac{0.71\ \varepsilon_{r(\text{eff})}}{\ln\left(\dfrac{\pi(d-w)}{w+t}+1\right)}\ \text{pF/in} \tag{2b}$$

If $d \gg w$, Eqs. (1), (2a), and (2b) can be further approximated as

$$\frac{C}{l} \approx \frac{\pi\varepsilon_{r(\text{eff})}\varepsilon_0}{\ln\left(\dfrac{\pi d}{w+t}\right)}\ \text{F/m} \tag{3}$$

$$= \frac{27.8\ \varepsilon_{r(\text{eff})}}{\ln\left(\dfrac{\pi d}{w+t}\right)}\ \text{pF/m} \tag{4a}$$

$$= \frac{0.7\ \varepsilon_{r(\text{eff})}}{\ln\left(\dfrac{\pi d}{w+t}\right)}\ \text{pF/in} \tag{4b}$$

For conductors of unequal widths, w_1 and w_2, the capacitance per unit length is given approximately as

$$\frac{C}{l} \approx \frac{2\pi\varepsilon_{r(\text{eff})}\varepsilon_0}{\ln\left[\pi^2 d^2\left(\dfrac{1}{w_1+t}\right)\left(\dfrac{1}{w_2+t}\right)\right]}\ \text{F/m} \tag{5}$$

$$= \frac{55.6\ \varepsilon_{r(\text{eff})}}{\ln\left[\pi^2 d^2\left(\dfrac{1}{w_1+t}\right)\left(\dfrac{1}{w_2+t}\right)\right]}\ \text{pF/m} \tag{6a}$$

$$= \frac{1.41\ \varepsilon_{r(\text{eff})}}{\ln\left[\pi^2 d^2\left(\dfrac{1}{w_1+t}\right)\left(\dfrac{1}{w_2+t}\right)\right]}\ \text{pF/in} \tag{6b}$$

If the ratio of the distance, d to the board thickness, h is $\gg 1$, $\varepsilon_{r(\text{eff})} \approx 1$. If $d \approx h$, $\varepsilon_{r(\text{eff})} = (1+\varepsilon_r)/2$ should be used.

Example 1:

Calculate the capacitance between two horizontal lands, similar to those shown in Fig. 2.13, located on a printed wiring board with a relative dielectric constant $\varepsilon_r = 4.5$. Calculate $\varepsilon_{r(\text{eff})}$ from measured data. The dimensions are

$t = 0.0028''$ (2 oz. copper)	$w = 0.025''$
$d = 1''$	$l = 6''$
$h = 0.060''$	$\varepsilon_r = 4.5$

With $d/h = 1''/0.060'' = 16.7 \gg 1$, $\varepsilon_{r(\text{eff})} \approx 1$, the capacitance is

$$C \approx \frac{0.7 \times \varepsilon_{r(\text{eff})} \times 6''}{\ln\left(\dfrac{\pi(1''-0.025'')}{0.025''+0.0028''}+1\right)}\ \text{pF}$$

$$= 0.904\ \varepsilon_{r(\text{eff})}\ \text{pF} \tag{7}$$

Measurements made for these dimensions yielded 0.924 pF. Thus,

$$\varepsilon_{r(\text{eff})} = \frac{0.942}{0.904} = 1.04 \tag{8}$$

Example 2:

Assume that the two traces used in Example 1 are moved closer together so that $d = 0.065''$. Here, $d/h = 0.065''/0.060'' = 1.1$.

$$C \approx \frac{0.7 \times \varepsilon_{r(\text{eff})} \times 6''}{\ln\left[\dfrac{\pi(0.065'' - 0.025'')}{0.025'' + 0.0028''} + 1\right]} \text{ pF}$$

$$= 2.49\, \varepsilon_{r(\text{eff})} \text{ pF } \textit{versus } 5.41 \text{ pF measured.} \qquad (9)$$

The measured effective dielectric constant in this case then, is

$$\varepsilon_{r(\text{eff})} = \frac{5.41}{2.49} = 2.17 \qquad (10)$$

Equation Development:

Equation (1) is adapted from Eq. (2), Formula Set C-1, where

$$\frac{C}{l} \approx \frac{\pi \varepsilon_{r(\text{eff})} \varepsilon_0}{\ln\left(\dfrac{d}{r}\right)} \text{ F/m}, \quad \text{for } \frac{2r}{d} \ll 1 \qquad (11)$$

By making the perimeter of the round conductor equal to that of the rectangular conductor we will get equal surface areas per unit length for the two geometries:

$$\text{Perimeter} = 2\pi r = 2(w + t) \qquad (12)$$

$$r \approx \frac{w + t}{\pi} \qquad (13)$$

Substituting r into Eq. (11) gives

$$\frac{C}{l} \approx \frac{\pi \varepsilon_{r(\text{eff})} \varepsilon_0}{\ln\left(\dfrac{\pi d}{w + t}\right)} \text{ F/m}, \quad \text{for } \frac{2(w + t)}{\pi d} \ll 1 \qquad (14)$$

The limitations of Eq. (14) are illustrated if we let $d = w$. Obviously, the equation is not valid when the conductors are touching and the capacitance per unit length becomes infinite. Adding a correction factor, Δ, to d so that we can get infinite capacitance when $d = w$:

$$\frac{C}{l} \approx \frac{\pi \varepsilon_{r(\text{eff})} \varepsilon_0}{\ln\left(\dfrac{\pi(d + \Delta)}{w + t}\right)} \text{ F/m}, \quad \text{for } \frac{2(w + t)}{\pi d} \ll 1 \qquad (15)$$

For infinite capacitance per unit length when $d = w$:

$$\ln\left(\frac{\pi(w + \Delta)}{w + t}\right) = 0 \qquad (16)$$

which means that

$$\left(\frac{\pi(w + \Delta)}{w + t}\right) = 1 \qquad (17)$$

Thus Δ is

$$\Delta = \frac{w + t}{\pi} - d \qquad (18)$$

Substituting Δ into Eq. (15) yields Eq. (1):

$$\frac{C}{l} \approx \frac{\pi \varepsilon_{r(\text{eff})} \varepsilon_0}{\ln\left(\dfrac{\pi(d - w)}{w + t} + 1\right)} \text{ F/m}$$

Please see Section 1.3, Electric Field Mapping, for a discussion on round *versus* rectangular conductors.

Equations (5), (6a), and (6b) for conductors of unequal width are derived from preceding Eqs. (11) and (14) as applied to Eq. (4) from Formula Set C-1.

Commentary and Conclusions:

1. EXP C-5A shows excellent correlation between predicted and measured capacitance values.
2. Because the capacitance is governed by the logarithmic term, distance (d) increases do not provide dramatic improvements. If, in Example 1, d is increased from 1" to 2", C decreases to 0.79 pF not by a factor of 2, as might intuitively be expected.
3. Sometimes, the geometric ratio d/h falls between approximately 1 and a number which is much greater than 1. If the application is critical, it is recommended that an effective dielectric constant, $\varepsilon_{r(\text{eff})}$, equal to $(1 + \varepsilon_r)/2$ be used. In some cases the conductors can be fully embedded in dielectric material as in multilayer circuit boards. In this case, the use of an effective dielectric constant, $\varepsilon_{r(\text{eff})} = \varepsilon_r$ should be considered.

2.2.6 Capacitance between a Flat Conductor and a Ground Plane, C-6

Ground planes can be used on circuit boards to provide large crosstalk reductions, as we will see later. This formula set gives the capacitance per unit length between a PWB land and the ground plane, as shown in Fig. 2.14.

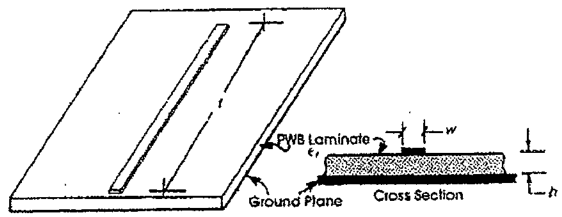

Figure 2.14 A PBW land is located a distance h above a ground plane.

This formula set introduces the relationship between the capacitive fringing factor, K_{C1}, and the inductive fringing factor, K_{L1}. If the medium surrounding the flat conductor and the ground plane is homogeneous, $K_{C1} = K_{L1}$. However, in this case, the flat conductor is separated from the ground plane by the PBW laminate with relative dielectric constant, ε_r. Because the region above the conductor is assumed to have a relative dielectric constant $\varepsilon_r = 1$, the medium surrounding the conductors is not homogeneous.

Note: The capacitance values found here are not to be confused with mutual capacitance.

Equations:

$$\frac{C}{l} = \varepsilon_r \varepsilon_0 K_{C1}\left(\frac{w}{h}\right)\text{F/m} \tag{1}$$

$$= 8.84\ \varepsilon_r K_{C1}\left(\frac{w}{h}\right)\text{pF/m} \tag{2a}$$

$$= 0.225\ \varepsilon_r K_{C1}\left(\frac{w}{h}\right)\text{pF/in} \tag{2b}$$

Example:

Assume that a 0.0156" wide land is 11" long and is separated from a ground plane by PWB laminate 0.060" thick with relative dielectric constant $\varepsilon_r = 4.5$. Compare this value with measured data.

Step 1: Determine $2h/w = 2 \times 0.060''/0.0156'' = 7.7$.

Step 2: Enter curve on Fig. 2.15 for $2h/w = 7.7$ and proceed upward to the fringing curve K_{C1}. Find $K_{C1} = 4.8$.

Step 3: Calculate capacitance from Eq. (2b).

$$C = 0.225 \times 4.5 \times 4.8 \times 0.0156''/0.060'' \times 11''$$

$$= 13.9\text{ pF } \textit{versus } 14.6\text{ pF measured} \tag{3}$$

Derivation:

The literature indicates that analytical solutions, for this deceptively simple problem, are difficult for the general case, where h is not small compared to w.

Reference [1] gives very accurate approximations for the characteristic impedance, Z_0, and the effective relative dielectric constant, $\varepsilon_{r(eff)}$, as a function of conductor width, w, relative dielectric constant, ε_r, and height, h, above a ground plane. These equations are

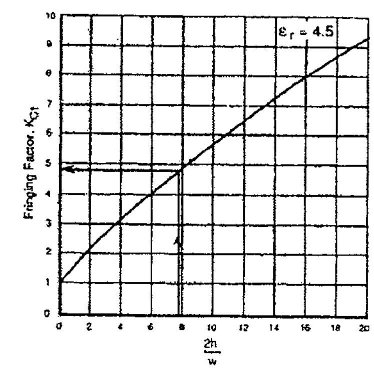

Figure 2.15 In a way similar to Formula Set C-4, the fringing factor, K_{C1}, is determined by entering the graph at the ratio $2h/w$, proceeding upward to the curve and locating K_{C1} on the left-hand vertical axis. Thus, for $2h/w = 7.7$, $K_{C1} = 4.8$.

For $\varepsilon_r = 1$:

$$Z_{0(\varepsilon_r=1)} \approx 60 \ln\left(\frac{8h}{w} + \frac{w}{4h}\right)\Omega, \qquad \frac{w}{h} \le 1 \tag{4}$$

$$Z_{0(\varepsilon_r=1)} \approx \frac{120\pi}{\dfrac{w}{h} + 2.42 - 0.44\left(\dfrac{h}{w}\right) + \left(1 - \dfrac{h}{w}\right)^6}\,\Omega, \qquad \frac{w}{h} \ge 1 \tag{5}$$

where

$Z_{0(\varepsilon_r=1)}$ = characteristic impedance with homogeneous medium and $\varepsilon_r = 1$.

For the relative dielectric constant, ε_r:

$$Z_{0(\varepsilon_r)} = \frac{Z_{0(\varepsilon_r=1)}}{\sqrt{\varepsilon_{r(eff)}}}\,\Omega \tag{6}$$

where

$Z_{0(\varepsilon_r)}$ = characteristic impedance with a dielectric with relative dielectric constant, ε_r, between the flat conductor and the ground plane;

$\varepsilon_{r(eff)}$ = effective relative dielectric constant:

$$= \frac{\varepsilon_r + 1}{2} + \left(\frac{\varepsilon_r - 1}{2}\right)\left(\frac{1}{\sqrt{1 + \dfrac{10h}{w}}}\right) \tag{7}$$

Note: $\varepsilon_{(eff)}$ in Ref. [1] is changed to $\varepsilon_{r(eff)}$ to be consistent with notation used in this book.

From Formula Set Z_0-(ALL), Z_0 is given by

$$Z_0 = \sqrt{\frac{L}{C}}\,\Omega \tag{8}$$

$$= \sqrt{\frac{L/l}{C/l}}\,\Omega \tag{9}$$

Formula Set L-6 and this formula set define L/l and C/l, respectively, as

$$\frac{L}{l} = \frac{\mu_r\mu_0}{K_{L1}}\left(\frac{h}{w}\right)\text{H/m} \tag{10}$$

$$\frac{C}{l} = \varepsilon_r\varepsilon_0 K_{C1}\left(\frac{w}{h}\right)\text{F/m} \tag{11}$$

where

K_{L1} = Inductive fringing factor, dimensionless
K_{C1} = Capacitive fringing factor, dimensionless

Combining Eqs. (9), (10) and (11),

$$Z_0 = \frac{1}{\sqrt{K_{L1}K_{C1}}}\sqrt{\frac{\mu_r\mu_0}{\varepsilon_r\varepsilon_0}}\left(\frac{h}{w}\right)\Omega \tag{12}$$

With $\mu_r = 1$, $\mu_0 = 4\pi \times 10^{-7}$ H/m and $\varepsilon_0 = 10^{-9}/36\pi$ F/m,

$$Z_{0(\varepsilon_r)} = \frac{120\pi}{\sqrt{K_{L1}K_{C1}}\sqrt{\varepsilon_r}}\left(\frac{h}{w}\right)\Omega \tag{13}$$

As previously stated, if $\varepsilon_r = 1$, the medium between the conductors is homogeneous and therefore $K_{L1} = K_{C1}$. Solving for K_{L1} by letting $Z_0 = Z_{0(\varepsilon_r=1)}$ in Eq. (13), we get

$$Z_{0(\varepsilon_r=1)} = \frac{120\pi}{K_{L1}\sqrt{1}}\left(\frac{h}{w}\right)\Omega \tag{14}$$

$$K_{L1} = \frac{120\pi}{Z_{0(\varepsilon_r=1)}}\left(\frac{h}{w}\right) \tag{15}$$

Note that K_{L1} is dependent only on the relative geometrical dimensions, h and w, and not on the dielectric constant of the material between the flat conductor and the ground plane. K_{C1} is, however, as will be seen in Eq. (16). We can solve for K_{C1} by combining Eqs. (6) and (13) and squaring

Table 2.2
Fringing Factors K_{L1}, K_{C1} and $Z_{0(\varepsilon_r=1)}$, $\varepsilon_{r(eff)}$ versus w/h ($\varepsilon_r = 4.5$)

$\frac{w}{h}$	$Z_{0(\varepsilon_r=1)}\Omega$	$\varepsilon_{r(eff)}$	$\frac{Z_{0(\varepsilon_r=1)}}{\sqrt{\varepsilon_{r(eff)}}}\Omega$	K_{L1}	K_{C1}	$\frac{2h}{w}$
0.100	262.94	2.92	153.76	14.33	9.30	20
0.125	249.56	2.94	145.44	12.08	7.90	16
0.2	221.41	3.00	127.94	8.51	5.68	10
0.25	208.06	3.02	119.66	7.25	4.86	8
0.5	166.82	3.13	94.27	4.52	3.14	4
1.0	126.51	3.28	69.88	2.98	2.17	2
2.5	78.69	3.53	41.87	1.92	1.50	0.8
5.0	49.64	3.76	25.60	1.52	1.27	0.4
10	29.21	3.99	14.62	1.29	1.14	0.2

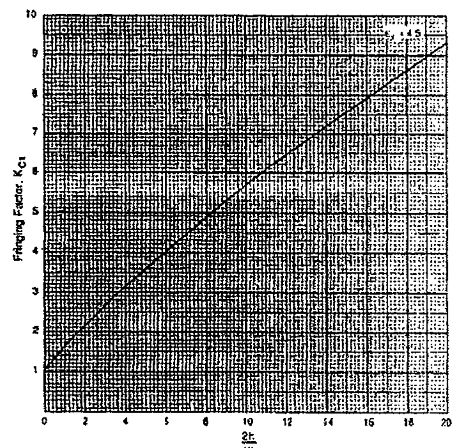

Figure 2.16 K_{C1} *versus* $2h/w$ with $\varepsilon_r = 4.5$. This curve indicates that, as would be expected, the fringing flux and hence the fringing factor increases with increasing height-to-width ratio, $2h/w$.

$$K_{C1} = \left[\frac{120\pi}{Z_{0(\varepsilon_r=1)}} \sqrt{\frac{\varepsilon_{r(eff)}}{K_{L1}\varepsilon_r}} \left(\frac{h}{w}\right)\right]^2 \qquad (16)$$

Table 2.2 lists $Z_{0(\varepsilon_r=1)}$, K_{L1} and K_{C1} for various values of w/h using $\varepsilon_r = 4.5$. K_{C1} for other values of ε_r can be determined in a similar manner.

Figure 2.16 plots the capacitive fringing factor, K_{C1}, *versus* the geometrical ratio $2h/w$. (K_{L1} is plotted in Formula Set L-6.)

The effective dielectric constant, $\varepsilon_{r(eff)}$ *versus* $2h/w$ is shown in Fig. 2.17.

In Fig. 2.12, we note that the ratio d/w is used for Formula Set C-4, Vertical Flat Conductors. Figure 2.18 illustrates the reason for this.

Commentary and Conclusions:

1. The solution developed above was easily applied to Formula Set C-4 because the electric fields are equivalent.

REFERENCES

1. Schneider, M. V., "Microstrip Lines for Microwave Integrated Circuits," *Bell System Technical Journal*, Vol. 48, No. 5 (May–June 1969).

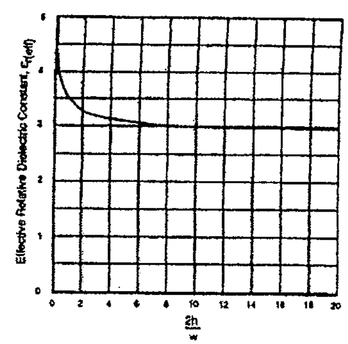

Figure 2.17 $\varepsilon_{r(eff)}$ *versus* $2h/w$ with $\varepsilon_r = 4.5$. As $2h/w$ approaches zero, $\varepsilon_{r(eff)}$ approaches 4.5, as would be expected because most of the electric flux is totally in the PWB laminate. Conversely, as $2h/w$ becomes large, $\varepsilon_{r(eff)}$ approaches 2.75, the average of the air and laminate dielectric constants.

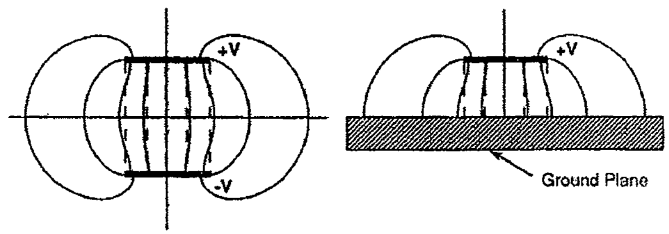

Figure 2.18 The flux pattern for the upper half of two vertical, flat conductors is identical to that for a single flat conductor over a ground plane. Thus, if we make $2h = d$, the correct fringing factor is found in Fig. 2.16.

2.2.7 Mutual Capacitance between Two Horizontal, Flat Conductors Near a Ground Plane, C-7

This is perhaps the most important of the capacitance formula sets because it applies directly to single-sided or multilayer printed wiring boards with a ground plane on one side or as a layer. This formula set shows the dramatic reduction in crosstalk provided by the addition of a ground plane. Figure 2.19 illustrates the geometry.

Equations:

The mutual capacitance/unit length is given approximately by Eq. (1):

$$\frac{C_m}{l} \approx \frac{\varepsilon_r \varepsilon_0}{\pi} K_{L1} K_{C1} \left(\frac{w}{d}\right)^2 \text{ F/m}, \quad \text{for } \frac{2h}{d} < 0.3 \tag{1}$$

$$= 2.81\ \varepsilon_r K_{L1} K_{C1} \left(\frac{w}{d}\right)^2 \text{ pF/m} \tag{2a}$$

$$= 0.07\ \varepsilon_r K_{L1} K_{C1} \left(\frac{w}{d}\right)^2 \text{ pF/in} \tag{2b}$$

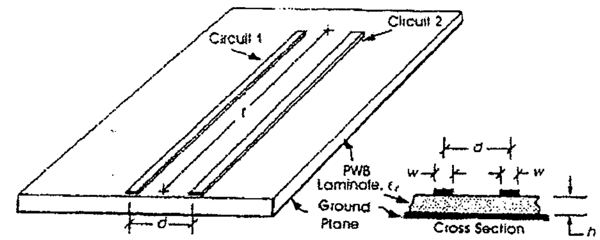

Figure 2.19 Two parallel lands are separated from a ground plane by the circuit board thickness, h.

Example:

Calculate the mutual capacitance between two PWB lands which are 0.025" wide, 1" apart, and 6" long on a 0.060" thick board with $\varepsilon_r = 4.5$.

Step 1: Check the ratio $2h/d$,

$$\frac{2h}{d} = \frac{2 \times 0.060''}{1''} = 0.12 \tag{3}$$

which is less than 0.3. If greater, Eq. (9) should be used.

Step 2: Determine the ratio $2h/d$

$$\frac{2h}{w} = \frac{2 \times 0.060''}{0.025''} = 4.8 \tag{4}$$

Step 3: Determine the fringing factors K_{L1} and K_{C1} from Fig. 2.20.

Step 4: Solving for C_m from Eq. (2b),

$$\frac{C_m}{l} = 0.07\ \varepsilon_r K_{L1} K_{C1} \left(\frac{w}{d}\right)^2 \text{ pF/in}$$

$$C_m = 0.07 \times 4.5 \times 5.1 \times 3.5 \times \left(\frac{0.025''}{1''}\right)^2 \times 6'' \text{ pF}$$

$$= 0.022 \text{ pF} \tag{5}$$

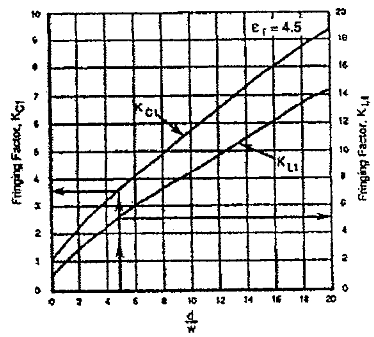

Figure 2.20 To find the fringing factors enter the curve at $2h/w = 4.8$ and find $K_{L1} = 5.1$ and $K_{C1} = 3.5$ on the vertical axes.

In the example of Formula Set C-5 for the same geometry, we had 0.90 pF. The improvement gained by adding the ground plane is a factor of 41.3, which would result in a crosstalk reduction of 32.3 dB. Please see EXP C-5 and EXP C-7 for additional details.

Derivation:

Formula Set C-3 gives the mutual capacitance between two circular conductors of equal height over a ground plane as

$$\frac{C_m}{l} = \frac{\left(\frac{L_m}{l}\right)\left(\frac{C_1}{l}\right)}{\left(\frac{L_1}{l}\right)} \tag{6}$$

where

$\frac{L_m}{l}$ = mutual inductance between conductors over a ground plane per unit length,

$\frac{C_1}{l}$ = capacitance between conductor 1 and the ground plane per unit length,

$\frac{L_1}{l}$ = self-inductance of conductor 1 and the ground plane per unit length.

L_m is found in Formula Set L-7 and is approximately

$$\frac{L_m}{l} \approx \frac{\mu_r\mu_0}{4\pi} \ln\left[1 + \left(\frac{2h}{d}\right)^2\right] \mathrm{H/m} \tag{7}$$

Formula Set C-6 gives the capacitance per unit length as

$$\frac{C}{l} = \varepsilon_r\varepsilon_0 K_{C1}\left(\frac{w}{h}\right) \mathrm{F/m} \tag{8}$$

From Formula Set L-6, the inductance per unit length is

$$\frac{L}{l} = \frac{\mu_r\mu_0}{K_{L1}}\left(\frac{h}{w}\right) \mathrm{H/m} \tag{9}$$

Combining Eqs. (6), (7), (8), and (9), we get

$$\frac{C_m}{l} = \frac{\frac{\mu_r\mu_0}{4\pi}\ln\left[1 + \left(\frac{2h}{d}\right)^2\right]\varepsilon_r\varepsilon_0 K_{C1}\left(\frac{w}{h}\right)}{\frac{\mu_r\mu_0}{K_{L1}}\left(\frac{h}{w}\right)} \mathrm{F/m}$$

$$= \frac{\varepsilon_r\varepsilon_0 K_{L1}K_{C1}\ln\left[1 + \left(\frac{2h}{d}\right)^2\right]}{4\pi\left(\frac{h}{w}\right)^2} \mathrm{F/m} \tag{10}$$

$$= \frac{\varepsilon_r\varepsilon_0}{4\pi} K_{L1}K_{C1}\left(\frac{w}{h}\right)^2 \ln\left[1 + \left(\frac{2h}{d}\right)^2\right] \mathrm{F/m} \tag{11}$$

Noting that $\ln(1 + x)$ can be expanded,

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \tag{12}$$

This series converges for $-1 < x \leq 1$. By neglecting all terms except the first and comparing the value of $\ln(1 + x)$ with x, we find the error to be 14% for $x = 0.3$, 5% for $x = 0.1$, *et cetera*. We can then simplify Eq. (11) by using Eq. (12) with $x = (2h/d)^2$. This procedure produces Eq. (1):

$$\frac{C_m}{l} = \frac{\varepsilon_r\varepsilon_0}{4\pi} K_{L1}K_{C1}\left(\frac{w}{h}\right)^2\left(\frac{2h}{d}\right)^2$$

$$= \frac{\varepsilon_r\varepsilon_0}{\pi} K_{L1}K_{C1}\left(\frac{w}{d}\right)^2 \text{ F/m} \qquad (13)$$

Commentary and Conclusions:

1. The example shows that a ground plane greatly reduces the mutual capacitance and hence crosstalk between two conductors.
2. As discussed in Formula Set C-3, the return power supply bus land is connected to the ground plane. Thus, the ground plane stays at zero potential with respect to other circuit land and component voltages.
3. Please note that the mutual capacitance is very distance sensitive. Decreasing the spacing by a factor of 10 increases the mutual capacitance by a factor of 100. Inspection of Eq. (1) indicates this is due to the squared term.
4. Subsection 5.2.2, Ground Plane Resistance (R-2) analyzes the "goodness" of the ground plane. The ground plane must be of sufficiently low impedance to prevent significant voltages being introduced between operational amplifier-summing junctions and the input signal.
5. The derivation of Eq. (1) is based, in part, on expressions for mutual inductance of circular conductors in which only the height above the ground plane, h, and the distance apart, d, are factors. In a way analogous to Formula Set L-7, the assumption is made that the total electric field potential distribution is approximately the same for flat conductors as it is for circular conductors. Thus, the potential at land 2 due to the voltage on land 1 will be about the same in either case.

2.2.8 Four-Conductor-System Mutual Capacitance, C-8

This formula set has application for determining the mutual capacitance between wiring cable pairs or other four conductor systems. The two circuits are assumed to be both ground, and mutually isolated.

Formula Set C-3 gave the value of the mutual capacitance between two conductors in the presence of a ground plane. This formula set solves for the mutual capacitance between two parallel line sets. Effects of shielding are neglected.

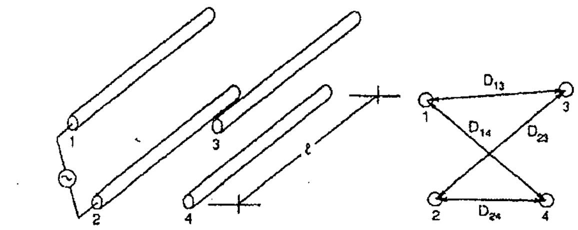

Figure 2.21 Four parallel conductors are spaced at arbitrary distances.

Figure 2.21 illustrates the geometry for two conductor pairs with a voltage impressed between conductors 1 and 2 and voltage pickup appearing on conductors 3 and 4.

Equations:

The mutual capacitance per unit length for the conductors shown in Fig. 2.21 (with conductor radii $r_1 = r_2$ and $r_3 = r_4$) is given for $D_{12} \gg 2r_1$, $D_{34} \gg 2r_3$ approximately as

$$\frac{C_m}{l} \approx \frac{\pi\varepsilon_{r(eff)}\varepsilon_0 \ln\left(\dfrac{D_{14} \times D_{23}}{D_{13} \times D_{24}}\right)}{2\ln\left(\dfrac{D_{12}}{r_1}\right)\ln\left(\dfrac{D_{34}}{r_3}\right)} \text{ F/m} \qquad (1)$$

$$= 13.9\, \varepsilon_{r(eff)} \frac{\ln\left(\dfrac{D_{14} \times D_{23}}{D_{13} \times D_{24}}\right)}{\ln\left(\dfrac{D_{12}}{r_1}\right)\ln\left(\dfrac{D_{34}}{r_3}\right)} \text{ pF/m} \qquad (2a)$$

$$= 0.35\, \varepsilon_{r(eff)} \frac{\ln\left(\dfrac{D_{14} \times D_{23}}{D_{13} \times D_{24}}\right)}{\ln\left(\dfrac{D_{12}}{r_1}\right)\ln\left(\dfrac{D_{34}}{r_3}\right)} \text{ pF/in} \qquad (2b)$$